ACCELERATION OF A CURRENT SHEET IN A DENSE GAS

A. K. Musin

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The motion of a current sheet in a rail-type plasma accelerator is examined in the case in which the dynamic resistance of the gas filling the accelerator is comparable with the electromagnetic forces. An estimate is made of the energy losses associated with the increase in the internal energy of the gas when a shock wave is formed, and possible conversion coefficients are discussed.

The dynamics of a current sheet in plasma accelerators of various types have been investigated by many authors (see, for instance, the comprehensive review [1]). In the majority of cases the conversion of the electric energy stored in the accelerating circuit into the kinetic energy of the accelerated plasma was investigated on the assumption that the resistance of the surrounding medium can be neglected as small in comparison with the electromagnetic forces of acceleration and retardation.

The authors of [2] took into consideration the effect of the viscous force of friction due to interaction of the particles diffusing from the current sheet with the guide electrodes and the accelerator walls, which causes inelastic losses and partial dissipation of momentum. The resistance of the undisturbed gas filling the accelerator was neglected as before. However, when a current sheet is accelerated in a plasma accelerator filled with a sufficiently dense gas, the energy dissipation due to collisions with gas molecules may become substantial and must be taken into consideration. In §1 the concept of a plasma piston raking up the gas in the accelerator is discussed, the conditions under which this concept is valid are examined, and a criterion is established for the sufficient gas density at which the dynamic resistance of the undisturbed gas is comparable to the electromagnetic forces acting on the current sheet. In §2, on the basis of these ideas, the energy-conversion coefficient is estimated for plasma accelerators operating in the plasma-piston mode.

§1. The plasma-piston concept. During gas breakdown in a plasma accelerator a current sheet is formed which under the influence of the self- or external-magnetic field moves along the guide electrodes, acting under certain conditions like an impenetrable plasma piston on the gas filling the accelerator. Ahead of this current sheet stretches a cloud of ionized gas whose existence (over distances of 10 cm and more) is maintained by spectral energy transfer from the main plasmoid as well as by electron diffusion processes and heat conduction (in the region $x \leq 1$ cm). The electron density in this plasma cloud varies from $n_e \sim 10^{15}$ cm^{-3} near the current sheet to $n_e \sim 10^{10}$ cm^{-3} at a distance $x \sim 15$ cm from it [18]. To entrain the charged particles effectively it is necessary that the time taken by the current sheet to cover a distance comparable with its thickness ($\tau_v \sim L/v$) be much less than the time τ_m taken by the magnetic field to penetrate the current sheet. The quantity τ_m can be estimated from the diffusion relation

$$L \sim (\tau_m \nu_m)^{1/2}, \quad \nu_m = c_0^2 / 4\pi\mu\sigma. \tag{1.1}$$

Here, ν_m is the magnetic diffusion coefficient (magnetic viscosity). Thus one of the conditions for the existence of a magnetic piston reduces to the inequality

$$\tau_m / \tau_v \sim Lv / \nu_m \gg 1, \tag{1.1'}$$

which coincides with the magnetohydrodynamic requirement $\mathrm{Re}_m \gg 1$ ($\mathrm{Re}_m = Lv/\nu_m$ is the magnetic Reynolds number). When this condition is satisfied, entrainment by the magnetic field predominates over penetration. For example, at $L \sim 1$ cm, $v \geq 10^7$ cm/sec we should have $\nu_m \leqslant 10^7$ cm^2/sec, i. e., the plasma conductivity $\sigma \geqslant 10^{13}$ sec^{-1} ~ 10 ohm^{-1} cm^{-1}. These conductivity values are easily achieved in plasma accelerators [18]. As far as the neutral atoms are concerned, if they are to be entrained by the moving current sheet as a result of charge-transfer processes and elastic collisions it is necessary that the effective flight path of the neutral atoms inside the current sheet $\lambda_a \sim [\langle n_i \rangle < S_{ai}(c_a) >]^{-1}$ not exceed its thickness L. In other words, the current sheet should have a sufficient piston density

$$P \equiv \langle n_i \rangle < S_{ai}(c_a) > L \gtrsim 1. \tag{1.2}$$

Here, $< S_{ai}(c_a) >$ is the cross section for momentum transfer from ionized atoms to neutrals averaged over the distribution function. For most gases $< S_{ai}(c_a) > \gtrsim 10^{-15}$ cm^2, and inequality (1.2) is satisfied if in a sheet of thickness $L \sim 1$ cm the average ion density $\langle n_i \rangle \gtrsim 10^{15}$ cm^{-3}. Such charged particle concentrations are normal for plasma accelerators [6,3].

All these conditions, however, while necessary, do not in fact guarantee effective entrainment of the gas by the current sheet. While traveling along the guide electrodes the current sheet may split into separate plasmoid filaments of thickness $\sim 10^{-1}$ cm, around which the gas flows easily, and the stationary plasma and neutral gas are not entrained.

The conditions for breakdown of the current sheet and the formation of filaments ("pinches"), which is one of the modes of instability of the current flow through a plasmoid, reduce to the relation between the magnetic self-field of the pinches and the corresponding external magnetic fields: the guide-electrode field, the field of the current sheets moving ahead of the pinches, etc. [5]. If the magnetic self-field of the pinches is small as compared to the external field, then pinches either do not form or rapidly disintegrate. The current sheet becomes more or less homogeneous, and the gas filling the accelerator is in fact entrained by the moving plasma.*

The concept of a plasma piston has been used in many experimental studies and the experimental results have been treated from this viewpoint, which is indirectly confirmed by the spatial separation of the ionization and luminescence fronts and by estimates of the degree of ionization in the current sheet (see, for example, [14–16]). Only in [6], however, which was undertaken precisely with this goal in mind, was it shown directly that in a rail-type accelerator a sheet of thickness ~ 1 cm can effectively entrain the gas filling the accelerator. It was discovered that in the current sheet there is a charged-particle concentrations gradient $\sim 3 \cdot 10^{17}$ cm^{-4}, and the plasma density turns out to be an order greater than it would be if the undisturbed gas were fully ionized. This testifies to effective raking of the gas by the current sheet under the conditions of the experiment.**

Let us calculate the momentum transferred per unit time by gas molecules of average density ρ_0 to a surface element of a current

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*The ionized gas may also be effectively entrained when the current sheet splits into separate filaments [17]. For this to happen it is necessary for the principal current to flow through the pinches, and for a condition of the (1.1) type to be satisfied with velocity $v = H_p(4\pi\rho_p)^{-1/2}$, where H_p is the pinch field, and ρ_p is the plasma density in the pinches. The mechanism of ionized-gas entrainment by the moving pinches is inductive in character [5].

**In [6] hydrogen at air initial pressure $p \sim 3 \cdot 10^2$ μ Hg was used as the working medium. The parameters of the accelerating circuit were: $C \sim 3\,\mu$F, $U_0 \sim 16$ kV, $i_{max} \sim 5 \cdot 10^4$ A. The electrodes were flat parallel plates. This geometry ensured a flat current sheet with a thickness of ~ 1 cm. The thickness of the current sheet was estimated from probe and optical measurements. The plasma density gradient was determined by the schlieren method, using the deflection of a light ray in the region with a refractive index gradient depending linearly on the charged-particle concentration.

sheet moving with velocity v. The gas-molecule velocity distribution in the coordinate system tied to the current sheet will be taken in the form

$$f(c_x; c_y; c_z) = n_0 (\pi v_0^2)^{-3/2} \exp\{-v_0^2 (c_x + v)^2 + c_y^2 + c_z^2]\},$$
$$v_0^2 = 2kT/m_a \qquad (1.3)$$

(the x-axis is directed along the outward normal to the surface of the sheet). In the immediate vicinity of the surface of the current sheet ($x < \lambda_a$) reflected particles slightly distort the initial distribution (1.3). The distribution of particles reflected from the current sheet depends on the scattering function w(c, c'), i. e., on the probability that as a result of collision with the sheet a particle will change its initial velocity c to c'. At the present time little is known about the function w(c, c'), and the perturbed distribution is not represented in analytic form (see (1.6)). However, at a sufficiently large distance from the sheet ($x \geqslant \lambda_a$) the distribution again approaches equilibrium. Therefore, to grasp the essential characteristics of the phenomenon we assume that the distribution function of the incident molecules is close to (1.3) [19]. The momentum transferred per unit time to a surface element of the current sheet by the impinging gas molecules with velocities from (c_x, c_y, and c_z) to ($c_x + dc_x$, $c_y + dc_y$, and $c_z + dc_z$) may be written as

$$d(\delta F_\rho) = -f(c_x; c_y; c_z) m_a c_x^2 dc_x dc_y dc_z. \qquad (1.4)$$

The total momentum δF_ρ transferred to a surface element is found after integrating (1.4) over the values of the velocity components $c_x \in [0, \infty)$, $c_y \in (-\infty, \infty)$, $c_z \in (-\infty, \infty)$; as a result we obtain

$$-\delta F_\rho = {}^1/_4 \rho_0 v_0^2 \left\{1 + \Phi_2\left(\frac{v}{v_0}\right)\right\} +$$
$$+ \frac{1}{\sqrt{\pi}} \rho_0 v v_0 \exp\left[-\left(\frac{v}{v_0}\right)^2\right] + {}^1/_2 \rho_0 v^2 \left\{1 + \Phi_0\left(\frac{v}{v_0}\right)\right\}$$
$$\Phi_0(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-x^2} dx, \quad \Phi_2(x) = \frac{4}{\sqrt{\pi}} \int_0^z x^2 e^{-x^2} dx,$$
$$v_0^2 = \frac{2kT}{m_a}. \qquad (1.5)$$

To calculate the total resistance of the stationary gas, it is necessary to take into account in (1.5) the momentum carried away by molecules reflected from the current sheet. This momentum depends on the character of the reflection, which, in turn, is determined by the physical nature of the interaction between the moving sheet and the stationary gas. In the case of elastic collisions, there may be specular reflection, when each molecule preserves the previous values of c_y and c_z, but the sign of c_x changes, and diffuse scattering with conservation of the absolute velocity, when the molecules are reflected at any angle with the same probability (for each velocity the number of reflected molecules in the solid angle $d\Omega$ is proportional to $d\Omega$). Only a small fraction of the molecules ($\leqslant 10^{-1}$) undergoes specular reflection [20].

As far as diffuse reflection is concerned, inelastic diffuse scattering with accommodation and in which there is partial energy transfer between the molecule and the sheet is physically more probable; the molecule is reflected, carrying away only part of its initial energy [22]. This process of inelastic scattering is characterized by the probability of a change in the velocity of the molecule on reflection, i.e., by the function w(c, c'). The flux of molecules scattered per unit surface of the sheet $\delta N' \equiv f(c') cx'$ is related to the incident particle flux $\delta N \equiv f(c) c_x$

$$f(c') c_x' = -\int_{c_x < 0} w(c, c') f(c) c_x dc \qquad (1.6)$$

which, in principle, affords a possibility of taking into account the momentum carried away by the reflected molecules. However, the processes of molecular scattering and absorption are complicated and insufficiently understood, and not much is known of the properties of the function w(c, c'). Therefore the inelastic-scattering mechanism is usually described [19, 21] by means of the effective accommodation coefficient $\beta = \varepsilon^{-1}\Delta\varepsilon = 1 - (c'/c)^2$, where $\Delta\varepsilon$ is the energy lost on the average by the reflected molecule. In this case the velocity of the reflected molecules on the average is $c' = c(1 - \beta)^{1/2}$. Taking this into consideration leads to the appearance of a correction factor $\alpha \sim 1 + (1 - \beta)^{1/2}$ on the right hand side of (1.5), which can be taken into account by introducing the "equivalent gas density" $\rho_\alpha = \alpha\rho_0$. For total absorption of the molecules $\beta = 1$, $\rho_\alpha = \rho_0$; elastic reflection when $\beta = 0$, leads to $\rho_\alpha = 2\rho_0$. In the general case the accommodation coefficient $\beta \in (0,1)$ and depends on the mass ratio of the gas molecules, and on the gas temperature in the layer and in front of it, as well as on other factors. On the average, we can assume that $\beta \sim \sim (1/2)(m_a/m_p)$, i.e., in our case, apparently, $\beta \sim 1/2$. Here, the coefficient $\alpha \sim 1.7$, i.e., $\rho_\alpha \approx 1.7\rho_0$. Thus, if we replace the initial gas density ρ_0 with the equivalent density ρ_α, expression (1.5) will determine the total resistance experienced by a surface element of the current sheet in the case of an arbitrary relation between the sheet velocity v and the most probable random velocity v_0 of the gas molecules.

In two limiting cases (1.5) simplifies.

(1) At small velocities, when $(v/v_0)^2 \ll 1$, the values of $\Phi_0(v/v_0)$ and $\Phi_2(v/v_0)$ are close to zero, and the resistance experienced by a surface element of the current sheet in raking up the undisturbed gas, may be represented as

$$-\delta F_\rho \approx \frac{\rho_\alpha v_0^2}{4}\left\{1 + \frac{4}{\sqrt{\pi}}\frac{v}{v_0}\left[1 - \left(\frac{v}{v_0}\right)^2\right]\right\} \approx \frac{\rho_\alpha v_0^2}{4} + \frac{\rho_\alpha v_0 v}{\sqrt{\pi}}. \qquad (1.7)$$

In the limit, as $v \to 0$, $\delta F_\rho \to -(1/4)\rho_\alpha v_0^2 = -(1/2) n_\alpha kT$, i.e., it is completely determined by the random motion of the molecules.

(2) At large velocities, when $(v/v_0)^2 \gg 1$ $\Phi_0(v/v_0)$ and $\Phi_2(v/v_0)$ differ little from unity, and for the momentum transferred to the sheet we can write

$$-\delta F_\rho \approx \frac{\rho_\alpha v_0^2}{2}\left[1 + \frac{2}{\sqrt{\pi}}\frac{v}{v_0}\exp-\left(\frac{v}{v_0}\right)^2\right] + \rho_\alpha v^2 \approx n_\alpha kT + \rho_\alpha v^2. \qquad (1.8)$$

At sufficiently large velocities ($v \to \infty$) the resistance is whooly determined by the directional velocity of the sheet v

$$\delta F_\rho \to -\rho_\alpha v^2. \qquad (1.9)$$

In the intermediate sheets it is necessary to use general formula (1.5) (replacing ρ_0 by ρ_α), which is convenient for analysis and calculations because of the possibility of using tabulated functions $\Phi_0(z)$ and $\Phi_2(z)$.

As a crtierion of the dynamic gas density we can use the ratio of the resistance force (1.5) to the electromagnetic forces of acceleration and retardation in the self- and external-magnetic fields—the gas should be considered sufficiently dense if this ratio is comparable with or exceeds unity. At sufficiently large velocities (case (2), $(v/v_0)^2 \gg \gg 1$) this condition is equivalent to the inequality

$$\min\left\{\frac{2c_0^2\rho_\alpha v^2 S^*}{i_{max}^2 l}; \frac{c_0\rho_\alpha v^2 S^*}{i_{max} Br}\right\} \gtrsim 1. \qquad (1.10)$$

Here, i_{max} is the maximum current in the sheet, c_0, the speeds of light, l denotes the increase in inductance per unit length of accelerator, B is the induction of the external magnetic field, r is the distance between the accelerating electrodes, and S^* is the effective surface of the current sheet. In plasma accelerators of the usual size with maximum currents $i_{max} \sim 3 \cdot 10^4$ A and the velocities $v \sim \sim 10^7$ cm/sec, condition (1.10) is satisfied at an equivalent gas density $\rho_\alpha \geqslant 3 \cdot 10^{-8}$ g/cm³. This means that hydrogen will be sufficiently dense at an initial pressure $p_0 \geqslant 1.5 \cdot 10^2$ μHg; in the case of air, the density criterion is satisfied at $p_0 \geqslant 10$ μHg; in the case of mercury vapor, at $p_0 \geqslant 1.5$ μHg (saturated vapor pressure at $T_0 \geqslant 20°$ C), etc.

§2. Energy conversion coefficient for plasma piston acceleration. When a current sheet moves in a dense gas, it is preceded by a shock wave, which moves away from it. At the shock front the parameters determining the state of the gas suffer a discontinuity.* A considerable portion of the energy is expended on compressing and heating the gas, i. e., on changing its internal energy. The remainder of the electromagnetic energy of the acceleration circuit is expended on accelerating the gas and is converted into the kinetic energy of the directional motion of the particle flux entrained by the plasma piston. As soon as the shock wave reaches the end of the guide electrodes and the gas begins to flow freely into the vacuum, an expansion wave (a simple Riemann wave) will travel through the gas compressed by the shock wave to meet the current sheet. When the expansion wave reaches the surface of the current sheet, a reflected wave is formed, and the subsequent motion of the gas is described by the general solution (see, for example, [9]). The problem is considerably simplified in the case of large velocities, when $M^2 = v^2/\gamma R_0 T_0 \gg 1$, where $v_{s0} = (\gamma R_0 T_0)^{1/2}$ is the speed of sound in the undisturbed gas ahead of the shock front. In this case a strong shock is formed, and the speed of sound behind the shock front is given by the expression

$$v_{s1}^2 = \gamma \frac{p_1}{\rho_1} = \frac{(1/2)(\gamma+1)\rho_0 v^2 \gamma}{\rho_0 (\gamma+1)/(\gamma-1)} = {}^1\!/_2\, \gamma(\gamma-1) v^2 \qquad (2.1)$$

At $\gamma < 2$ the speed of sound behind the shock front is less than the velocity of the current sheet, with which moves the disturbed gas raked up by the plasma piston. Therefore the Riemann expansion wave moving through the compressed gas to meet the current sheet is unable to reach its surface, and the entire acceleration process ends before the reflected wave has a chance to form.

The thermodynamic parameters of the disturbed gas behind the shock front (subscript 1) and the velocity v_w of the shock wave can be determined at sufficiently large current sheet velocities (strong shock) from the following known relations [9]:

$$\frac{\rho_1}{\rho_0} = \frac{\gamma+1}{\gamma-1}, \qquad v_w = \frac{1}{2}(\gamma+1)v. \qquad (2.2)$$

We assume that everywhere ahead of the shock front, the gas parameters correspond to the undisturbed state, i. e., we neglect the change in the state of the gas ahead of the shock front owing to energy transport by diffusing electrons and plasma radiation from the current sheet.** As the energy-conversion coefficient we take the ratio of the kinetic energy of directional motion of the gas raked up by the plasma piston and the accelerated material from the electrodes and the walls of the accelerator to the work done by the electromagnetic forces in accelerating the current sheet and overcoming the resistance of the undisturbed gas on the acceleration section:

$$\eta = \left(1 + \frac{W_0}{W_1 + W_1'}\right)^{-1}$$

$$W_0(t) = \int_0^x \rho_\alpha v^2(t)\,dx,$$

$$W_1(t) = \frac{1}{2}\int_x^{x_w} \rho_1 v^2(t)\,dx, \qquad W_1'(t) = \frac{1}{2}\int_x^{x_w} \rho_1' v^2(t)\,dx. \qquad (2.3)$$

* The shock wave formed when plasma is electrodynamically accelerated was experimentally observed, for instance, in [7, 8]. The pressure of the undisturbed gas was $p_0 \geq 0.7$ mm Hg which, according to (1.10), ensured a sufficient gas density. Separation of the luminescence and ionization fronts, which can be related to the "contact surface" (current sheet) and the shock front, respectively, and motion of the fronts at various speeds were also observed in [14, 15].

** Evidently, in energy calculations, which are in the nature of estimates, this is admissible, since the energy expended on the ionization, excitation, and dissociation of the gas ahead of the shock front does not exceed a few percent of the kinetic energy of the accelerated plasma. Therefore, the subsequently calculated energy-conversion coefficient can be considered as a fairly close upper limit.

Here $W_0(t)$ is the work done in overcoming the resistance of the gas filling the accelerator, $W_1(t)$ is the kinetic energy of the disturbed gas moving ahead of the current sheet $W_1'(t)$ is the kinetic energy of the accelerated material released from the electrodes and the accelerator walls as a result of ion bombardment and Joule heating [2], and $x(t)$ and $x_w(t)$ are the linear coordinates of the current sheet and the shock front, respectively.

For the cases usually encountered in experimental practice, when the initial voltages and capacitances are not too large, i. e., when the following condition is satisfied

$$\frac{l^2 C U_0}{2c_0^2 L_0 K(1+\chi)} \lesssim 1, \qquad (2.4)$$

the rather complicated expressions for the capacitor voltage $U(t)$ and the velocity $v(t)$ of the current sheet [10] can be approximated by simple relations of the form [11]:

$$U(t) = U_0 \exp\left(-\frac{t}{2\tau_*}\right)\cos\omega t,\ v(t) =$$

$$= \frac{lU_0}{2c_0^2 RK(1+\chi)}\left[1 - \exp\left(-\frac{t}{\tau_*}\right)\right],$$

$$\tau_* = (CR\omega^2)^{-1}, \quad \omega^2 = c_0^2/CL_0 - (Rc_0^2/2L_0)^2,$$

$$\chi = m_0/KCU_0. \qquad (2.5)$$

Here, U_0 is the initial capacitor voltage, τ_1 is the effective relaxation time of the acceleration process, ω is the equivalent frequency, c_0 is the speed of light; L_0 is the initial inductance of the accelerating circuit, C is the capacitance, R is the total ohmic resistance, l is the increase in inductance per unit length of the accelerating electrodes, K is the erosion factor (gas-discharge electrochemical equivalent of the electrode material [10]), χ is a parameter characterizing the intensity of electrode erosion in the acceleration process: when $\chi \gg 1$, the role of erosion in the formation of the current sheet is large, when $\chi \ll 1$ the role of erosion is small [2, 11]. At $l \sim 3$ henry/cm $L_0 \sim$ $\sim 3 \cdot 10^2$ cm, $K \sim 10^{-14}$ g/CGSE charge unit [12], and when $\chi \ll 1$ inequality (2.4) is satisfied if $C \leq 10^8$ cm $\sim 10^2$ μF, $U_0 \leq 60$ CGSE pot. units $\sim 2 \cdot 10^4$ V. In these circumstances for instants between the first and second extrema of the voltage, the mass $m(t)$ of material removed from the electrodes, which is a function of the current $i(t)$ in the sheet, will obey the law

$$m(t) = K\int_0^t |i(\tau)|\,d\tau \approx \frac{2}{\pi} U_0 K C \omega t. \qquad (2.6)$$

With these assumptions the W_k are calculated and after simple transformations for the energy conversion coefficient of the plasma accelerator we obtain

$$\eta(x) = \left\{1 + \frac{E_0}{(1/4)(\gamma+1)E_1 + R_v E_2}\right\}^{-1},$$

$$E_0 = \frac{x}{\lambda} - 3\left(1 - \exp\frac{-x}{\lambda}\right) +$$

$$+ \frac{3}{2}\left(1 - \exp\frac{-2x}{\lambda}\right) - \frac{1}{3}\left(1 - \exp\frac{-3x}{\lambda}\right),$$

$$E_1 = \frac{x}{\lambda} - \frac{6}{\gamma-1}\exp\frac{-x}{\lambda}\left\{1 - \exp\left[-\left(\frac{\gamma-1}{2}\right)\frac{x}{\lambda}\right]\right\} +$$

$$+ \frac{3}{\gamma-1}\exp\frac{-2x}{\lambda}\left[1 - \exp\frac{-(\gamma-1)x}{\lambda}\right] -$$

$$- \frac{2\exp\frac{-3x}{\lambda}}{3(\gamma-1)}\left[1 - \exp\frac{-3(\gamma-1)x}{2\lambda}\right],$$

$$E_2 = \frac{x}{\lambda} + \frac{1}{\gamma-1}\exp\frac{-2x}{\lambda}\left[1 - \exp\frac{-(\gamma-1)x}{\lambda}\right] -$$

$$- \frac{4}{\gamma-1}\exp\frac{-x}{\lambda}\left[1 - \exp\frac{-(\gamma-1)x}{2\lambda}\right],$$

$$R_v = \frac{v_*}{v_\infty}, \quad v_* = \frac{U_0 CK\omega}{\rho_\alpha \pi S^*}, \quad v_\infty = \frac{lU_0}{2c_0^2 RK(1+\chi)},$$

$$\lambda = \tau_* v_\infty, \quad \tau_*^{-1} = CR\omega^2.$$

The quantity $\eta(x)$ approaches 1 as $x \to 0 (x/\lambda \ll 1)$ and decreases monotonically with increase in x. As $x \to \infty (x/\lambda \gg 1)$ the conversion coefficient asymptotically approaches its limiting value

$$\eta_{\lim} = \left\{1 + \frac{1}{(1/4)(\gamma+1) + R_v}\right\}^{-1},$$

$$R_v = (2c_0^2/\pi) CK^2 \omega R (1+\chi) / \rho_\alpha S^* l. \tag{2.8}$$

This limiting value of the conversion coefficient increases with γ and depends strongly on the ratio R_V (see Figs. 1 and 2).*

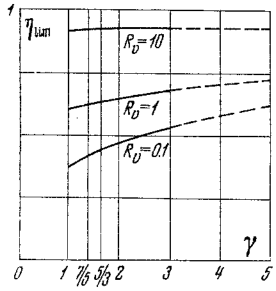

Fig. 1. Limiting energy conversion coefficient $\eta_{\lim}$ as a function of γ.

In particular, it follows that in polyatomic gases the values of the limiting conversion coefficient are smaller than in monatomic gases. This is because in the case of complex molecules a considerable portion of the electromagnetic energy is unproductively expended on the internal degrees of freedom.

The rate of change of the energy conversion coefficient depends importantly on the effective relaxation length $\lambda = \tau_* v_\infty$. At small values of λ the conversion coefficient $\eta(x)$ reaches values close to $\eta_{\lim}$ even at comparatively small x. Figure 3 illustrates the function $\eta = f(x\lambda)$; γ and R_V are parameters.

In modern plasma accelerators, and for real gases, the energy conversion coefficient in the "plasma piston" mode does not exceed ~0.5. In fact, if for estimating purposes we take an initial voltage $U_0 \sim 3 \cdot 10^3$ V, capacitance $C \sim 10^2$ μF, total resistance $R \in (1, 10)$ ohm, effective frequency $\omega \sim 10^5 \text{sec}^{-1}$, erosion coefficient $K \sim \sim 10^{-14}$ g/CGSE unit of charge, and an inductance increment per unit length of the electrodes $l \in (1, 10)$ henry/cm, then $R_V \leqslant 10^{-1}$, and the effective relaxation length $\lambda \in (10^{-1}, 1)$ cm. This means that for ordinary plasma accelerators of length $x \in (10, 10^2)$ cm at a ratio of specific heats $j \in (1.1, 3)$ the energy conversion coefficient $\eta \in (0.2, 0.5)$.

1. When an accelerator operates in the "plasma piston" mode under conditions actually realizable at the present time, the coefficients of conversion of the electromagnetic energy stored in the accelerating circuit into the kinetic energy of the accelerated plasma are relatively small, since a large portion of the energy is expended on increasing the internal energy of the gas in the formation of a shock wave ($\eta \lesssim 0.5$).

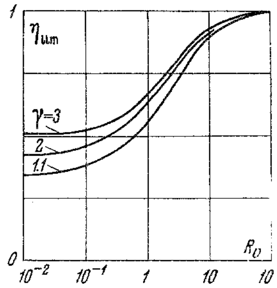

Fig. 2. Limiting energy conversion coefficient $\eta_{\lim}$ as a function of R_V.

2. The energy conversion coefficient is a decreasing function of the electrode length that asymptotically approaches some limiting value depending on γ and R_V. The limiting values of the conversion coefficient are reached the more rapidly, (at smaller accelerator lenghts), the smaller the inductance increment and the initial capacitor voltage, and the larger the capacitance, the total resistance, and the electrode erosion coefficient.

3. When monatomic gases are accelerated, higher values of the energy conversion coefficient can be reached than in the case of polyatomic gases, since in the latter case a significant portion of the energy is expended on the internal degrees of freedom.

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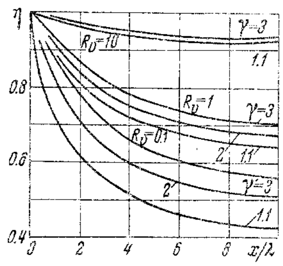

Fig. 3. Energy conversion coefficient η as a function of the relative length x/λ.

*In Fig. 1 the dashed lines correspond to values of the adiabatic exponent $\gamma > 3$. Under normal conditions γ does not exceed 3 ($\gamma = (k+2)/k$, where k is the number of degrees of freedom of the gas molecule), $\gamma = 3$ corresponding to a directional gas flow moving at high velocity as a continuum with one degree of freedom. However, in describing the relations between the thermodynamic gas parameters with allowance for electronic and molecular excitation, dissociation, and ionization, we can introduce an effective adiabatic exponent γ such that the true adiabatic curve which, generally speaking, does not coincide with the Poisson curve, approximates it in the neighborhood of a given point. In this case values of $\gamma > 3$ may correspond to actual equations of state [13].

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